# RECENT PUBLICATIONS.

## REVIEWS.

*First Course in the Theory of Equations.* By L. E. DICKSON. New York, John Wiley and Sons, 1922. 8vo. 6 + 168 pages. Price, $1.75.

Elementary algebra like elementary physics is less a single subject than an aggregate of distinct sciences loosely related by analogy of material and method. Even when the scope is restricted as here to the theory of equations, there need be no obvious continuity in the range of topics covered. The textbook under review is devoted to the customary questions together with a few important additions. The particular sequence followed is satisfactory but perhaps no better than any one of a dozen other possible arrangements of the same material. There is an unevenness in the book that is inherent in any such aggregation of separate studies. Some chapters are concrete to the extent that details of technique will occupy most of the student's attention, while other portions are so abstract that only the author's careful exposition and fortunate selection of proofs for exercises saves the subject from being entirely beyond the reach of the elementary student. Just what is to be regarded as elementary and what as advanced is perhaps more a question of the manner and style of exposition and the character of the previous preparation of the student than an inherent feature of the subject itself. For example, certain treatises on the calculus are surely beyond the grasp of the college freshman, while other treatments are perhaps ideal material for first-year work at institutions of high grade. Part of the difficulty that many of the less alert students are sure to find in a course based upon this or any similar book lies in the very instability of the problem, if one may venture so harsh a figure. No sooner has one point of view been attained than an entirely new line of investigation demands a change of attitude. The reviewer feels that many plodding students would find it easier to follow a suitable, connected course in even such a subject as the Galois theory, regularly regarded as advanced, than a course such as this which is more traditional but less unified. He would raise the question as to whether the investigation of numerical approximations to the roots of an algebraic equation here featured does not fit more readily into a course on numerical methods (a course which might develop the theory of least squares, interpolation, and numerical integration) than into a discussion devoted to such items as the representation of any symmetric function in terms of the elementary symmetric functions, and the impossibility of trisecting a general angle under the usual restrictions on method. However, traditional reasons will render this particular selection of subject matter acceptable to many teachers who may not be enthusiastic over the new material but who would miss any topic now included had it been omitted from this but not from older works.

The chapter headings[1] give a fair idea of material embraced in this text, but

[1] I: Complex numbers, 1–10; II: Elementary theorems on the roots of an equation, 11–28; III: Constructions with ruler and compasses, 29–44; IV: Cubic and quartic equations; their discriminants, 45–54; V: The graph of an equation, 55–70; VI: Isolation of the real roots of a real equation, 71–85; VII: Solution of numerical equations, 86–100; VIII: Determinants, systems of linear equations, 101–127; IX: Symmetric functions, 128–142; X: Elimination, resultants and discriminants, 143–154; Appendix: The fundamental theorem of algebra, 155–158.

fail to suggest the surprising richness of content and elegance of proof that mark the entire work. It is seldom that an American author succeeds in incorporating so much solid information in an elementary text of this size. This has been accomplished by careful discussions, judicious illustrative examples, and a relegation of all subsidiary material to exercises. The exercises in fact constitute a significant part of the work. Few of them are extremely hard and none are trivial. They may come as a shock to students fed on the sort of drill so common in freshman texts consisting of nothing but substituting numbers in literal formulas. They will require much more class-room discussion but the hints that are given should bring them within the range of well-prepared students.

Instructors who have used the author's *Elementary Theory of Equations* with undergraduates will find much familiar material here. They will note some corrections (for example, the author here defines the minor of a general determinant, while in the other book the definition was only to be inferred from a special case), but the most grateful feature is the revision and simplification of the subject matter and the very greatly enriched discussions. The work is indeed distinctly simplified and without any serious loss in completeness. Most of the interesting technical details that have been omitted for the sake of simplification are referred to in footnotes.

Fledglings are notoriously incapable of long flights. Any discussion which requires the student's consecutive attention for more than a page of distinct logical steps unrelieved by illustration or summary, exhausts and almost eliminates most students and is pedagogically indefensible. The individual items may be clear and simple but a many-linked chain of reasoning may easily drag down a student and can seldom support him. The most serious pedagogical criticism of the book lies in the failure to provide breathing spaces in one or two of the more extended discussions (for example, in paragraph 31, "Cubic equations with a constructible root"). Complicated statements of theorems tend to be meaningless for most students, but this difficulty is usually inherent in the subject, and the manner of exposition is not to be criticized. (Compare, page 81, "If the discriminant $\Delta$ of $z^4 + qz^2 + rz + s = 0$ is negative, there are two distinct real roots and two imaginary roots; if $\Delta > 0$, $q < 0$, $L > 0$, four distinct real roots; if $\Delta > 0$ and either $q \geqq 0$ or $L \leqq 0$, no real roots." Here $L = 8qs - 2q^3 - 9r^2$.)

But whatever the objections one may raise, there is no doubt that the book can be used with profit in classes with sophomores. The poorest students cannot fail to acquire much important and interesting information, and good students will have their eyes opened to many fruitful fields of mathematical thought. At all stages the book hints of other unexplored territories and is as suggestive an incitement to the ambitious beginner as could be asked. There is no need to dwell upon the logical neatness with which the author usually succeeds in saying exactly what he intends. All through the subject of algebra there are scattered pitfalls for the unwary and few texts are not guilty of laxity if not of deliberate error in the statement of definitions or theorems. The most delightful element

of this book is that the statements made do not require modification by implicit restrictions to be inferred from the context. The more fundamental the notions used, the more satisfactory is the clearcut language. We need more texts like this in America, books that can be used in every college, but which represent from elementary aspects the finest of mathematical scholarship.

A few minor infelicities may be mentioned: A complex number is discussed under two distinct representations. The less common is called explicitly (page 3) "the trigonometric form," while the other remains unnamed. Immediately after emphasizing the fact that a complex number has many amplitudes differing by integral multiples of 360°, the term, "the amplitude" is used without a hint as to which one is "the amplitude." The terms, "linear factor" and "factored form" are defined in the quadratic case, while the terms are then employed in connection with the general polynomial without further discussion. A product constituting a *polynomial* is called the factored form of an *equation*. In connection with mathematical induction the author speaks of "changing $n$ into $n + 1$." The phrase, "upper limit to the real roots" is defined as a single concept without any inquiry as to the meaning of the word, "limit," or the sense of "upper," but in the exercises, the words "lower limit" are mentioned casually as though then familiar to the reader. "The numbers (10) are known as *Cardan's Formulas*." The discriminant, after being defined in general on page 47, is redefined for the quartic on page 51. "A point on the graph at which the tangent is both horizontal and an ordinary tangent is a bend point. . . ." In the exercises on page 74 the letters must denote only real quantities although no such warning is given. But minor blemishes such as these and rare misprints do not seriously impair the usefulness of a simple but scholarly text.

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*Mathematical Philosophy. A study of fate and freedom. Lectures for educated laymen.* By CASSIUS J. KEYSER. New York, E. P. Dutton & Company, 1922. 8vo. 14 + 466 pages. Price $4.70.

In the preface of this book its author expresses the hope that these lectures may not be ungrateful to the following two classes of readers, among others:

"To the growing class of such professional mathematicians as are not without interest in the philosophical aspects of their science. To the growing class of such teachers of mathematics as endeavor to make the spirit of their subject dominate its technique."

The lectures were designed primarily for students whose major interest is in philosophy, but the present review is restricted to a consideration of the merits of the book for the two classes of readers just noted. Such readers will find in this volume much that is inspiring, much that they will enjoy to re-read, much that will be instructive and will lead them to look at subjects from a new point of view. Comparatively few of these readers will probably find here the enduring qualities of real mathematics, but they will find certain views which will extend their horizon as regards the nature and bearing of real mathematics and which will enable them to present their subject in a more popular form.